Engineering Analysis of the Deformation of Fencing Swords

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The topic of deformation (compression and bending) of SCA fencing weapons comes up from time to time. In October, 2011, there was a discussion on this topic on the SCA's society level mailing list for Kingdom Rapier Marshals. I made a long post describing the engineering equations for each of the types of deflections we were talking about. Since I expect that the topic will continue to come up, I've copied, expanded, and updated that post for possible future use.

First, a small introduction: In the SCA, I'm a former Kingdom Rapier Marshal for Atlantia who has also lived and marshaled in the Midrealm and West. Modernly, I have a Ph.D. in Aerospace Engineering and do structural design, analysis, and sizing for aircraft and rockets for NASA.

Three types of deflection are discussed in this article. The first is axial compression, where the blade gets shorter due to a compressive force. The second is cantilever bending, where the applied force is perpendicular to the blade causing it to deflect off axis. The third type is axial buckling, where an applied axial force eventually causes the blade to suddenly pop from being straight to being curved.

Following the theory sections, I have some discussion on my thoughts and recommendations on the issue.

Axial Compression

A long steel rod acts as an extremely stiff spring. If you pull on it, it will get longer. If you push on it, it will get shorter. You may never have noticed this. As I'll show you, there's a good reason why.

Symbols:
E - Modulus of Elasticity (Young's modulus). Units are lb/in^2 (psi)(or pascals in SI)
ε (epsilon) - Strain, change in length divided by original length. Unit less (inches per inch)
σ (sigma) - Stress, intensity of loading. Units are lb/in^2 (psi) (or pascals in SI)
F - Applied load, pounds
A - Cross sectional area, in^2

δ (delta) - change in length, inches

Equations:

$\sigma = E\,\varepsilon$ (Hooke's Law)

replace

$\sigma = F/A$

$\varepsilon = \delta/L$

and rearrange to get an equation for the axial deflection of the blade tip under axial load:

$$\boldsymbol{\delta = \frac{F\,L}{E\,A}} \quad \text{(Equation 1)}$$

Plug in your blade length and cross section (L and A), the force you're applying (F), and the modulus for steel (E, about 30×10^6 psi) and you'll see that the typical compression (δ) due to loading (F) is immeasurably small outside of a lab.

For our purposes, there is effectively no pure axial compression of these blades. What we're really after is bending.

<u>Cantilever Bending</u>

The current SCA blade stiffness test is checking cantilever bending. We're holding the grip horizontally and applying a weight to push the blade down vertically. This test does not exactly match how the sword is used and deflects in combat, but as I will show is a reasonable proxy test for what we're doing.

The equation for the lateral deflection of the tip in this case is:

$$\boldsymbol{\delta = \frac{F\,L^3}{3\,E\,I}} \quad \text{(Equation 2)}$$

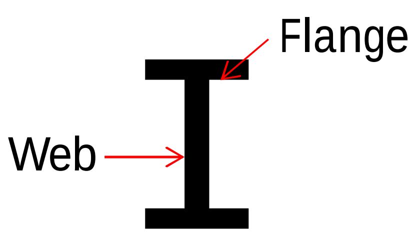
F, L, and E were defined above.

"I" is the moment of inertia. It is a function of the cross sectional shape. As you can see from the equation, a higher value is more resistant to bending than a lower value. Units are in^4

Computing I for an arbitrary shape takes calculus or a computer:

$$I = \int_A h^2 \, dA$$

“h” is the distance from the bending axis of the cross section. The more material that is at a large distance from the bending axis of the cross section, the higher the moment of inertia, I, is. An “I-Beam” has a high moment of inertia because most of its material is out in the flanges (where the value of “h” is high”). They are used when high bending stiffness is desired.



I-Beam Cross Section

Consider a typical SCA blade. When you bend the blade by pushing on the flat side it takes *much* less force than bending the blade by pushing sideways on the edge. This is because the moment of inertia measures the bending resistance of the blade shape. A short, wide oval or diamond has a much lower value for I than does the tall, narrow oval/diamond that is the same blade at 90 degrees.

Before your eyes roll up in your head any further, we have charts giving the value for I for a large selection of common shapes.

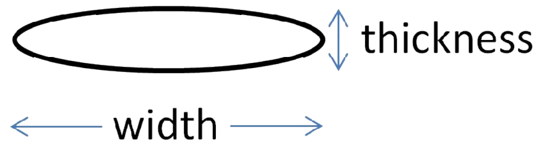
The one that seems closest for our purposes is the ellipse:

$$\mathbf{I = \pi \, a \, b^3 / 4}$$

where

π = 3.14159....

a and **b** are the two radii of the ellipse (half the diameters). In our case I'd use "a" as half the blade width and "b" as half the blade thickness. If you swap the values of “a” and “b” you’ll get the moment of inertia for the blade rotated 90 degrees.



Oval Blade Cross Section

You could also back out the value for I by performing a bending test with a known load. And, if you play with the numbers in equation 2 you can see, unlike the axial compression case, a measurable deflection happens for a small load. This is as we expect and observe.

The current SCA blade bending test uses a known weight (F) and verifies that it produces a minimum required deflection (δ) . It is, therefore, measuring the remaining terms in equation 2, i.e. L^3 / 3 E I. Note also the strong, cubed, effect of blade length, L, on the deflection. The same cross section shape will bend much more on a 45” blade than on a 35” one.

<u>Axial Buckling</u>

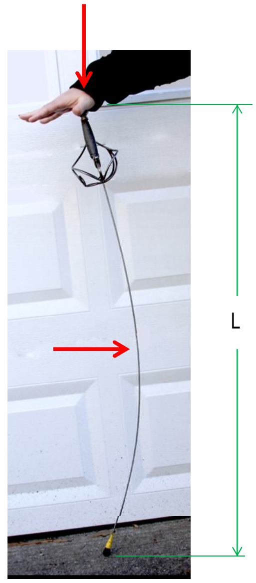
Buckling is a more complex phenomenon. In our case, picture putting the tip on the ground and pushing straight down on the pommel. At some reasonably small load, the blade will suddenly pop out of line and the pommel will move down. To me, buckling is more representative of our actual SCA use of the blade and how hard it is possible to strike someone with that blade.

There are very similar equations for this type of deformation which is also called column buckling. But, the equations are very strongly dependent on the “boundary conditions”. Boundary conditions are how the ends of the blade/column are being held/not held. If both ends are free to rotate (engineers say pinned) you get a very different value than if one or both ends are not free to rotate (engineer: fixed). In English, you'll get a much higher buckling load if you grip the handle rather than palming the pommel. If we end up designing a test based on buckling we'll have to understand that and be very clear in our instructions.

Pinned-Pinned Buckling (both ends can rotate - palm on pommel)

$$\boldsymbol{P} = \frac{\boldsymbol{\pi^2 E\, I}}{\boldsymbol{L^2}} \quad \text{(Equation 3)}$$

Where π, E, I, and L were defined before and **P** is the critical buckling load.

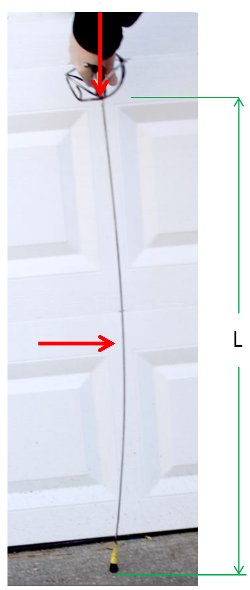


Example of “Pinned-Pinned”: Palm on Pommel, Grip can rotate, use Eqn. 3

Pinned-Fixed (tip can rotate, grip held to not rotate)

$$\boldsymbol{P} = \frac{\mathbf{2.04}\,\boldsymbol{\pi^2 E\, I}}{\boldsymbol{L^2}} \quad \text{(Equation 4)}$$

The buckling load is twice as high here and the length of the column is lower (you can't count the portion that is being held), so what you'd measure on the scale would be more than twice as high for the same blade.



Example of “Pinned-Fixed”: Hand on Grip, Grip cannot rotate, use Eqn. 4

If would be fairly easy to pick a desired maximum buckling load that could be field tested using a portable scale (using the palm on pommel option reduces the scale maximum weight requirement so may be preferable as it would allow cheaper/smaller scales). It would take some data gathering to determine what that maximum pass/fail value should be. A very quick check of my day-to-day blade suggests that the fail line would be in the general ballpark of 20 pounds. Amazon sells a variety of portable bathroom scales for about $20. They all seem to require batteries.

Discussion

Equation wise, the main difference between the status-quo bending weight test and the buckling test is the "L" term. In equation 2, L is cubed. In equations 3 and 4 it is squared. What that means is that a long blade will appear to be more stiff if we switch standards to the buckling test. The two approaches both measure "E I" in the same way and will produce the same ranking of blades when tested.

Practicality wise, how often do we check blade stiffness? How much more difficult is requiring a portable scale (with working batteries) over a fishing weight? Does switching gain us anything? Are there bad blades that pass the current test that the new test could eliminate?

My bottom line feeling is that while a buckling test would more accurately model the real world use of our blades, the bending test that we're currently using is a more than adequate proxy test. It measures the same things but in a different way. It is cheaper and more practical to perform in the field. Unless some evidence appears that indicates the buckling test is capable of discriminating a blade we don't want on the field better than the bending test the extra effort does not seem worthwhile to me.